Math 591: Topological Combinatorics Fall 2019

Instructor: Prof. Patricia Hersh
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Office: 3226 SAS Hall
Office Hours: Tuesdays 10:30-11:30am, Wednesdays 4-5pm and by appointment.

Class Meeting Time and Location: MW 1:30-2:45pm, Daniels Hall 232.

Text: Rather than a textbook, this course will draw material from the following survey articles, many of which are freely available at their authors' web pages. Students should not hesitate to contact the instructor if they need assistance obtaining any of these articles.

1. Poset topology: tools and applications, by Michelle Wachs, available for free at http://www.math.miami.edu/ and as a chapter in the book "Geometric combinatorics", pages 497–615, IAS/Park City Math. Ser. 13, Amer. Math. Soc., Providence, RI, 2007

2. An introduction to hyperplane arrangements, by Richard Stanley, available for free at http://www-math.mit.edu/ rstan/ and as a chapter in the book "Geometric combinatorics", pages 389–496, IAS/Park City Math. Ser. 13, Amer. Math. Soc., Providence, RI, 2007.

3. Topological methods in Combinatorics, by Anders Björner, pages 1819–1872 in the book "Handbook of Combinatorics, Volume II" edited by Ronald Graham, Martin Grotschel, and Laszlo Lovasz.

4. A user's guide to discrete Morse theory, by Robin Forman, freely available at: http://www.math.rice.edu/ forman/

5. Subspace arrangements, by Anders Björner, chapter in the book "First European Congress of Mathematics, Vol. I (Paris, 1992)", pages 321–370, Progr. Math. 119, Birkhauser, Basel, 1994.

6. The homology and shellabilty of matroids and geometric lattices, by Anders Björner, chapter in the book "Matroid applications", pages 226–283, Encylopedia Math Appl., 40, Cambridge Univ. Press, Cambridge, 1992.

Prerequisites: MA 551/753 or permission of instructor. It would also be helpful to have taken or be concurrently enrolled in MA 524/724.

Course Overview and Topics: poset topology, discrete Morse theory, shellability, group actions and group representations on posets, hyperplane arrangements and subspace arrangements, geometric lattices and matroids, face numbers of polytopes and simplicial complexes. The exact list of topics and amount of emphasis placed on the topics will depend somewhat

on the interests of the students enrolled. In addition to discussing theoretical topics, the course will include a few very compelling applications to computer science, for instance to complexity theory lower bounds obtained by calculating Möbius functions.

Learning Outcomes: Students should gain familiarity with techniques of topological combinatorics, particularly regarding partially ordered sets and poset topology. They should also gain increased skill in proving results in this area and increased familiarity with the literature in this area. The professor will also try to help students make the jump to being independent researchers by giving advice throughout the semester regarding how to carry out research, using the course topics to help illustrate how research results were achieved by others in the past and how this area developed over time with different researchers in this field contributing distinct perspectives that each added to the picture of what is known.

Course Structure: The course will center around lectures. There will be a small amount of homework with students given many weeks to complete each assignment and a lot of choice as to which problems to solve from a list of possibilities, designed to encourage exploration of topics the students find most interesting and the development of increased independence of a somewhat similar style to what is needed in independent research.

Course Schedule: Chapter on hyperplane arrangements by Richard Stanley (3-4 weeks); Quick review of representation theory (1-2 weeks); Chapter on poset topology by Michelle Wachs (4-5 weeks); Chapter on topological combinatorics by Anders Björner (2 weeks); Discrete Morse theory (2 weeks). This schedule is subject to change.

Homework: There will be recommended problems, a few of which will be collected.

Grading: Course grades will be determined by homework (100 percent of grade).

Late policy: Late homework will not be accepted. This specifically includes late arrival to class on the day when homework is due.

Attendance Policy: Regular attendance is expected and active participation is encouraged. Please let the instructor know if you must miss class. Please do not hesitate to ask questions in class. It helps everyone! Please see the University's Attendance Regulation (Reg02.20.07) for further information.

Academic Integrity Statement: Students are expected to indicate clearly in their homework any references they use to assist them and also indicate when their work is collaborative with other class members. Collaboration is encouraged, but should be acknowledged. Please see the Code of Student Conduct Policy (NCSU POL 11.35.1) for further information.

Statement for students with disabilities: "Reasonable accommodations will be made for students with verifiable disabilities. In order to take advantage of available accommoda-

tions, students must register with the Disability Resources Office at Holmes Hall, Suite 304, 2751 Cates Avenue, Campus Box 7509, 919-515-7653. For more information on NC State's policy on working with students with disabilities, please see the Academic Accommodations for Students with Disabilities Regulation (REG02.20.01)"

N.C. State University Policies: "Students are responsible for reviewing the PRR's which pertain to their course rights and responsibilities. These include:

https://policies.ncsu.edu/policy/pol-04-25-05 (Equal Opportunity and Non-Discrimination Policy Statement),

https://oied.ncsu.edu/oied/policies.php (Office for Institutional Equity and Diversity),

https://policies.ncsu.edu/policy/pol-11-35-01 (Code of Student Conduct), and

https://policies.ncsu.edu/regulation/reg-02-50-03 (Grades and Grade Point Average).